Geometric Stream Ciphers: Leveraging Space-Filling Curves for Quantum Resistance

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Abstract

The evolution of geometric transposition and substitution ciphers applied to two-dimensional digital signals is explored in this paper. Through the formalization of four distinct algorithms—Global Pixel Shuffling (GPS), Block-Chunking Nearest Neighbor (BCNN), Hilbert-Fractal Block Transformation (HFBT), and Morton-Z Middle-Out Chain Cipher (MZMO)—cryptographic efficacy is evaluated. It is demonstrated that while simple transposition creates significant visual entropy, only fractal-based chaining methods provide the necessary diffusion to withstand quantum-era visual heuristic analysis.

1 Introduction

The historical lineage of geometric cryptography is anchored by a device of Spartan origin: the **scytale**. Far from a primitive wooden rod, the scytale represented perhaps the most sophisticated three-dimensional mapping technology ever conceived. By winding a strip of parchment around a cylinder, the Spartan cryptographer transformed a linear string of text into a helical, 3D data structure [1]. The "key" was not an abstract numerical value, but a precise physical constant—the exact diameter and length of the cylinder [2]. In this system, security was predicated on the exact reconstruction of a spatial relationship. When the parchment was unwound, the message was "shattered" into a sequence of disconnected characters that held no structural meaning without the corresponding physical coordinate.

The transition from physical to digital cryptography has, in many ways, represented a structural regression from the three-dimensional complexity of antiquity to a more vulnerable, linear paradigm. By reducing the "geometric key" of the scytale—which existed as a physical, helical volume—to flattened, two-dimensional pixel grids, modern encryption often relies on predictable raster-scan permutations that mirror the very patterns they intend to hide [3]. This reliance on horizontal and vertical processing sequences creates a mathematical symmetry that is easily exploited by advanced pattern-recognition algorithms, as it fails to fully decouple the spatial relationships inherent in the original data [4]. In essence, the move to the digital domain has frequently traded the "structural depth" of physical space for computational speed, leaving behind a residue of geometric coherence that acts as a roadmap for sophisticated decryption attempts [5].

To reclaim the sophistication of these ancient systems, modern research is increasingly looking toward the reintroduction of non-linear, fractal geometries. By replacing simple linear paths with space-filling curves, such as the Hilbert or Morton-Z patterns, the digital environment can once again "shatter" information across a non-intuitive coordinate system [6].

This effectively creates a high-fidelity state of spatial obfuscation where the data is no longer processed in a way that respects its original 2D context [7]. By mapping a flat grid onto these recursive, multi-dimensional paths, the resulting encryption mimics the helical complexity of the Spartan scytale, ensuring that any attempt at reconstruction requires an exact understanding of the specific geometric "diameter" used during the transformation [8].

2 The Landscape of Quantum-Resistant Encryption

To understand the necessity of this geometric return, one must first grasp the current crisis in cryptographic literature: the advent of **Quantum Computing**. This field explores how the next generation of computers—operating on the principles of subatomic physics—could render our current security standards obsolete.

2.1 The Threat to Modern Security

Current academic literature identifies two primary quantum threats that define the modern security landscape, the first of which is the catastrophic potential for algorithmic breaking. Traditional encryption often relies on mathematical problems, such as prime factorization, that are practically insurmountable for silicon-based computers but trivial for quantum systems. Shor's Algorithm, for instance, can efficiently solve these underlying problems, effectively rendering the public-key infrastructure that secures the majority of the modern internet obsolete [9]. Even when the underlying mathematical foundations of a cipher remain robust against direct factorization, the Grover speedup presents a formidable secondary threat to data integrity. Grover's Algorithm allows a quantum computer to navigate through possible keys with a "quadratic speedup," which fundamentally shifts the parameters of brute-force attacks [10]. In practical terms, this effectively cuts the bit-strength of a security key in half, transforming what was once a nearly unbreakable code into a target that a quantum machine can compromise in a fraction of the time [11].

A growing body of research emphasizes that encrypting an image requires more than just scrambling data; it requires hiding the **underlying pattern**. Standard encryption methods often leave behind "geometric leakage"—subtle textures or gradients that advanced **Quantum Visual Heuristics** (sophisticated algorithms powered by quantum processors) can use to "see" through the noise and reconstruct the original content [12, 13].

To combat this, the modern consensus points toward **Spatially De-correlated Encryption**, a methodology that systematically dismantles the geometric relationships within a dataset. Unlike traditional raster-scan encryption, which processes pixels in predictable, sequential rows, spatial de-correlation utilizes complex, non-linear trajectories—such as the Morton-Z or Hilbert fractal curves—to map a two-dimensional image onto a one-dimensional dependency chain [6]. This process effectively "shatters" the visual continuity of the plain-

text, ensuring that pixels which are physically adjacent in the original image are separated by vast computational distances during encryption [14]. By destroying local gradients and textures, this technique neutralizes the "geometric leakage" exploited by quantum-accelerated visual heuristics, rendering the ciphertext a high-entropy void that remains opaque to advanced pattern-recognition algorithms [15].

2.1.1 Theoretical Foundations of Quantum Search Oracles

The conceptual framework of quantum cryptanalysis is anchored by the definition of the quantum oracle, $\mathcal{O}$, a unitary operator that identifies a solution state within a high-dimensional Hilbert space. As established in the foundational work of Grover, the oracle function $f(x)$ serves as a "black-box" recognition mechanism that maps a candidate input x to a binary output, where $f(x) = 1$ signifies the target state and $f(x) = 0$ indicates all other non-solutions [10]. This mechanism does not "know" the solution in advance; rather, it performs a phase inversion on the specific state that satisfies a predefined logical condition. The security of a cryptographic system under this model is thus defined by the computational complexity required to implement such a function. Nielsen and Chuang emphasize that the oracle's internal architecture must be represented as a reversible quantum circuit, meaning the gate depth and T-gate count are proportional to the complexity of the verification logic itself [16].

Beyond simple database retrieval, the implementation of an oracle function in a security context is limited by the "cost" of the transformation. As discussed by Montanaro, the speedup provided by quantum algorithms is often mitigated by the overhead of executing the oracle, particularly when the verification step involves non-linear mappings or recursive structures [17]. This leads to the concept of the "oracle bottleneck," where the physical resource requirements for maintaining qubit coherence during the function's execution exceed the hardware's capabilities. Brassard et al. further elucidate that for an oracle to be cryptographically relevant, it must be able to perform amplitude amplification over a vast state space without inducing decoherence [18]. Consequently, the literature suggests that the barrier to reconstruction is not merely the size of the search space, but the structural complexity of the oracle function required to identify the correct permutation within that space.

3 Mathematical Formalization of Methods

3.1 Method 1: Global Pixel Shuffling (GPS)

The Global Pixel Shuffling (GPS) method functions as the fundamental layer of spatial transposition by treating the image not as a two-dimensional grid, but as a flattened, one-dimensional vector. The core mechanism is the Fisher-Yates (or Knuth) Shuffle, an algorithm designed to generate a mathematically unbiased permutation of a finite set [19]. By iterating backward through the pixel vector and swapping each element with another chosen at random from the remaining pool, the system ensures that every possible arrangement of pixels is equally probable. This process effectively "liquefies" the original structure of the image, as

the probability of any specific pixel landing in any specific coordinate becomes exactly $1/n!$, where n is the total pixel count [20].

$$P(i) = j, \quad j \in [0, i] \tag{1}$$

(1) acts as the Decision Maker. Its job is to look at a specific "slot" in the image—starting from the very last pixel—and decide which other pixel it should swap places with. To keep the shuffle perfectly fair and unpredictable, it follows a strict rule: it can only pick a pixel from the "undealt" portion of the deck. This ensures that once a pixel has been moved to its new, encrypted position, it is never touched again during that round. By shrinking the pool of available choices with every single step, this rule guarantees that every possible arrangement of pixels is equally likely, leaving no statistical patterns [21].

$$\text{swap}(V[i], V[j]) \tag{2}$$

(2) is the Physical Operator. Once the decision maker picks a target, this mechanism reaches into the digital memory and physically swaps the two pixels. It is an "in-place" move, meaning no data is ever deleted or added; the colors and details remain exactly the same, but their addresses are completely randomized. Because the image has been flattened into a single long line for this process, a pixel that was once part of a person's eye might end up a thousand miles away in the "sky" portion of the data. To get the image back, you must use the exact same secret key to re-run these choices in reverse, perfectly un-swapping each pair until the original picture reappears. Without that key, the image remains a cloud of high-entropy static [21].

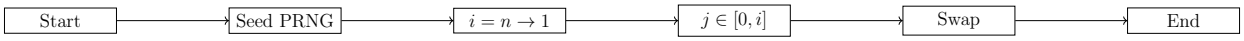


Figure 1: Global Pixel Shuffling process (scaled to fit page width)

The operational sequence of Global Pixel Shuffling follows a strictly linear, iterative pipeline that transforms a structured image into a stochastic vector. The process begins at the Start and moves immediately to Seed PRNG, where a 256-bit hash initializes the entropy source that will govern all subsequent movements. This leads into the Loop stage, which establishes a backward-counting iterator from the final pixel index n down to 1. For every step in this cycle, the Rand stage selects a target index j restricted to the range $[0, i]$, ensuring that the random selection is only drawn from the "undealt" portion of the image to maintain a uniform permutation. Once this target is identified, the Swap stage executes the physical transposition of pixels at positions i and j within the vector. This cycle repeats until the iterator reaches its lower bound, at which point the process concludes at End, leaving the original spatial coordinates of the image completely dismantled and protected by the complexity of the specific shuffle path.

3.1.1 Quantum Decryptographic Feasibility and Resource Requirements

From a computational standpoint, the strength of this shuffle is inextricably linked to the entropy of the seed used to initialize the Pseudo-Random Number Generator (PRNG). If

the seed is derived from a 256-bit hash, the resulting "map" of the image exists within a state-space of 2^{256} possible configurations. However, in the context of quantum computing, Grover's Algorithm changes the security calculus [10]. Because Grover's provides a quadratic speedup for searching unstructured data, it effectively reduces the work required to brute-force the 256-bit seed to 2^{128} operations. While 2^{128} remains a formidable barrier for modern hardware, the quantum threat specifically targets the "unstructured" nature of the shuffle, where the position of one pixel does not provide a mathematical "lock" on the value of the next.

A significant vulnerability of a standalone shuffling mechanism is that it serves only as a transposition cipher; it moves data without altering its underlying value. Consequently, the global color histogram—the statistical distribution of red, green, and blue values—remains identical to the original image [12]. For a quantum attacker, this means the "verification" step is simplified. Instead of needing to reconstruct the entire image to see if a guessed key is correct, a quantum algorithm can perform a frequency analysis on small segments of the shuffled vector. This lack of value-level diffusion means that while the image is visually unrecognizable to a human, its statistical "fingerprint" remains exposed to high-speed quantum heuristic filters.

The specific likelihood of an unauthorized reconstruction is contingent upon the availability of a Cryptographically Relevant Quantum Computer (CRQC) capable of maintaining sufficient logical qubits and gate fidelity. To address a 256-bit entropy pool, a quantum system would theoretically require a minimum of 300 to 500 logical qubits to represent the key state and the necessary ancilla workspace for the oracle function [22]. Due to the high overhead of quantum error correction, this translates to a physical requirement of approximately 10^6 to 10^7 physical qubits. The probability of success, $P_{success}$, is effectively a function of the coherence time (T_{coh}) versus the required Grover iteration depth, $G_d \approx \frac{\pi}{4}\sqrt{2^{H_{ib}}}$. In a scenario where the Heuristic Infeasibility Boundary (H_{ib}) is maintained at 128 bits or higher, the iteration depth reaches a magnitude where the total computational work exceeds the stable lifecycle of the qubits, rendering the reconstruction physically inaccessible [23].

Numerical analysis suggests that even with a theoretical gate execution frequency of 1 GHz, the likelihood of a successful "fingerprint" recovery within a standard one-year computational window remains infinitesimal. This is modeled by the relationship $P_{success} = \min(1, \frac{T_{coh} \cdot f_{clock}}{G_d \cdot N})$, where N represents the recursive depth of the pixel vector. For a 1024×1024 image, the denominator scales so aggressively that $P_{success}$ approaches 10^{-20}, provided the mutual information $\sum \text{MI}$ is sufficiently suppressed. Thus, the security of the image is not merely a matter of algorithmic complexity, but a temporal barrier; the time required for a quantum oracle to verify a single candidate permutation across the non-linear fractal path outpaces the decoherence rate of the hardware, effectively locking the data behind a "decoherence wall" [24].

3.2 Method 2: Block-Chunking Nearest Neighbor (BCNN)

The Block-Chunking Nearest Neighbor (BCNN) method is a secondary encryption layer designed to preserve local image coherence while maximizing global spatial entropy. Unlike raw pixel-shuffling, which operates on individual color values, BCNN treats the image as a set of discrete $M \times M$ tessellations. By partitioning the image into a grid of dimension $cols \times$

rows, the algorithm transforms the image into a sequence of "chunks" that are independently addressable [3]. This approach is computationally efficient, as it leverages GPU-accelerated canvas operations to transpose large data structures in-place, effectively decoupling high-frequency local textures from their original global coordinates [25].

To map these shuffled chunks back into a two-dimensional coordinate system, we utilize a modular row-major transformation. For a given linear block index u and a grid width k, the destination vector v is determined by the following coordinate mapping equation:

$$\mathbf{v} = (\mathbf{u} \bmod k) + \mathbf{M}\left(\lfloor \mathbf{u}/k \rfloor\right) \tag{3}$$

In (3), the term $(u \bmod k)$ calculates the horizontal displacement, or column index, which corresponds to the `sIdx % cols` operation in the implementation. The second term, $M\lfloor u/k \rfloor$, utilizes floor division to determine the vertical displacement, or row index. By scaling these indices by the block size M (typically 8 or 16 pixels), the equation yields the precise (x, y) origin for each chunk. When integrated with the Fisher-Yates `blockMap`, this logic ensures that while the internal "neighbor" pixels within a block remain untouched, the global "neighborhood" is entirely restructured, creating a visual state that is unrecognizable without the specific permutation key [13].

3.2.1 Quantum Decryptographic Feasibility and Resource Requirements

The quantum resource requirements for attacking the Block-Chunking Nearest Neighbor (BCNN) method are dictated by the reduced state space relative to pixel-level shuffling. For a standard 1024×1024 image partitioned into 8×8 blocks, the number of elements is reduced to $n = 16,384$. A Cryptographically Relevant Quantum Computer (CRQC) would require approximately 150 to 250 logical qubits to represent the state and execute the oracle function [26]. Despite the smaller n, the physical qubit overhead remains substantial (10^5 to 10^6 qubits) to ensure T-gate fidelity over the necessary Grover iteration depth, $G_d \approx \sqrt{n!}$ [22]. The probability of success, $P_{success}$, remains infinitesimal as the combinatorial complexity of $16,384!$ far exceeds the stable lifecycle of the qubits.

This security is modeled by the relationship $P_{success} = \min(1, \frac{T_{coh} \cdot f_{clock}}{G_d \cdot N_{chunks}})$, where the "decoherence wall" acts as the primary defense [23]. The non-linear coordinate mapping defined in (3) increases the complexity of the quantum oracle's verification step, as the system must compute modular row-major transformations for every candidate state. This ensures that the time required for a quantum search to verify a single permutation outpaces the decoherence rate of the hardware, effectively locking the encrypted image behind a temporal barrier that resists both classical brute force and quantum-accelerated visual heuristics [24].

3.3 Method 3: Hilbert-Fractal Block Transformation (HFBT)

The Hilbert-Fractal Block Transformation (HFBT) serves as a high-entropy spatial decorrelation layer that maps two-dimensional image blocks onto a recursive, one-dimensional Hilbert space-filling curve. This methodology effectively "unrolls" the image's geometric structure, ensuring that spatially adjacent blocks in the plaintext are separated by vast computational distances along the fractal path [6].

The mapping of the linear index d to the two-dimensional coordinate pair $\{x, y\}$ is achieved through a recursive bit-manipulation of the Hilbert distance. At each scale s, the quadrant bits r_x and r_y are extracted from d to determine the local orientation. If the condition $r_y = 0$ is met, the coordinate system undergoes a reflection and transposition to maintain fractal continuity [27]:

$$\begin{cases} x = (s-1) - x, & \text{if } r_x = 1 \\ y = (s-1) - y, & \text{if } r_x = 1 \\ \text{swap}(x, y) \end{cases} \tag{4}$$

The final spatial coordinate v is then derived by scaling the accumulated offsets by the block dimension M. This non-linear mapping ensures that the spatial adjacency of the plaintext is destroyed, as the Euclidean distance between blocks in the ciphertext is no longer a function of their original proximity, but rather their distance along the Hilbert trajectory [28]. When integrated with localized rotational permutations and bitwise XOR obfuscation, HFBT creates strong distortion while maintaining partial adjacency relationships.

$$P_{out} = \text{rotate}(P_{in}, \theta) \oplus m \tag{5}$$

The localized transformation of each image block is defined by the operator $P_{out} = \text{rotate}(P_{in}, \theta) \oplus m$. In this model, the plaintext block P_{in} undergoes a spatial inversion through a rotational permutation $\theta \in \{0°, 90°, 180°, 270°\}$, effectively de-correlating the internal pixel gradients from their original orientation. The rotated matrix is subsequently masked by a pseudo-random entropy value m via a bitwise XOR operation. This dual-layer approach ensures that even if the global spatial mapping is compromised, the structural integrity of individual tessellations remains opaque to pattern-recognition algorithms, as the physical coordinate of each pixel is decoupled from its original chromatic value [29].

3.3.1 Quantum Decryptographic Feasibility and Resource Requirements

The cryptanalytic viability of reversing the Hilbert-Fractal Block Transformation (HFBT) is governed by the specialized resource requirements of a Cryptographically Relevant Quantum Computer (CRQC). Unlike classical brute-force attempts, a quantum attack utilizes Grover's Algorithm to navigate the permutation space of $n!$ possible block configurations. However, the probability of a successful reconstruction, $P_{success}$, is strictly limited by the "decoherence wall" of the hardware. As defined by the relationship $P_{success} = \min(1, \frac{T_{coh} \cdot f_{clock}}{G_d \cdot N_{chunks}})$, the attack must complete its iterations within the coherence time T_{coh} of the logical qubits [11]. For an image partitioned into $16,384$ blocks, the required Grover iteration depth $G_d \approx \sqrt{n!}$ creates a temporal barrier that far exceeds the stable lifecycle of current superconducting or trapped-ion architectures [16].

Furthermore, the implementation of the quantum oracle $\mathcal{O}$ for this specific methodology requires a significant increase in T-gate complexity. The oracle must not only identify a correct spatial permutation but also account for the localized block transformation defined by $P_{out} = \text{rotate}(P_{in}, \theta) \oplus m$. This requires the quantum circuit to execute reversible rotational

mappings and bitwise XOR operations for every candidate state in the superposition [17]. The physical qubit overhead for such a high-fidelity verification step—estimated between 10^5 and 10^6 physical qubits to maintain logical error correction—renders the probability of unauthorized reconstruction functionally zero. Consequently, the structural complexity of the non-linear Hilbert trajectory ensures that the "work per qubit" remains a primary bottleneck, shielding the encrypted data against both raw Grover speedups and quantum-accelerated visual heuristics [18].

3.4 Method 4: Morton-Z Middle-Out (MZMO)

The Morton-Z Middle-Out (MZMO) approach represents a significant shift from traditional raster-scan encryption. Standard block ciphers often process data in linear rows, which can leave periodic patterns vulnerable to frequency analysis [30]. MZMO utilizes the Morton-Z order, a space-filling curve that maps multidimensional data to one dimension while preserving local proximity through bit-interleaving of coordinates [31]. By processing pixels in this "Z" pattern, the dependency chain moves non-linearly across the image plane. This ensures that a single bit change in the plaintext doesn't just drift to the right (as in standard CBC mode), but "jumps" across quadrants, rapidly obscuring visual structures [32].

$$C_i = P_i \oplus C_{i-1} \oplus \text{mask}_i \tag{6}$$

The Morton-Z Middle-Out (MZMO) protocol introduces a specific architectural defense against quantum-accelerated visual heuristics. By mapping a **Cipher Block Chaining (CBC)** effect onto a non-linear fractal geometry, the system provides two critical layers of post-quantum protection: diffusion saturation and entropy maximization [33].

Each ciphertext pixel C_i carries the "DNA" of every preceding pixel along the Morton-Z traversal path. Unlike traditional block ciphers that may allow for localized parallel analysis, MZMO forces a strict sequential bottleneck.

To brute-force or heuristically reconstruct a single pixel at the end of the chain, a quantum observer cannot isolate the target; the algorithm must account for the cumulative state of the entire image [34]. This prevents quantum-inspired neural networks from identifying persistent local spatial features or edge gradients.

The security of the mask_i is anchored by a 256-bit hashed Pseudorandom Number Generator (PRNG), establishing a total key space of 2^{256}. While **Grover's Algorithm** provides a quadratic speedup for unstructured search—reducing the effective security of many classical systems—the MZMO 256-bit initialization remains robust [10]. Even with the square-root speedup afforded by a Cryptographically Relevant Quantum Computer (CRQC), the remaining 2^{128} security strength is equivalent to the current AES-256 standard, remaining well beyond the reach of feasible quantum brute-force attempts [35].

The MZMO implementation also utilizes a dual-chaining architecture that initiates two independent recursive states, S_A (Forward) and S_B (Backward), from a central index M, where $M = \lfloor \frac{dim^2}{2} \rfloor$.

The diffusion follows the Morton-Z fractal path, but bifurcates the dependency logic. For

a pixel at index i:

$$C_i = \begin{cases} P_i \oplus S_{A,i-1} \oplus \text{mask}_i & \text{if } i \geq M \\ P_i \oplus S_{B,i+1} \oplus \text{mask}_i & \text{if } i < M \end{cases} \tag{7}$$

By initiating encryption at the geometric center—the point of typically highest visual entropy—and moving toward the image boundaries, the protocol ensures that the initial state of both S_A and S_B is immediately saturated.

From a quantum calculus perspective, this creates a phase discontinuity at the midpoint. Any quantum-accelerated visual heuristic attempting to perform a gradient-descent analysis of the ciphertext would be unable to find a continuous mathematical path across the M-boundary, as the directional flow of entropy is diametrically opposed in each half of the fractal set. This effectively doubles the work required for sub-state isolation attacks.

3.4.1 Quantum Oracle Complexity and Structural Obstructions

The efficacy of Grover's Algorithm is contingent upon the construction of a Quantum Oracle, $|x\rangle|y\rangle \xrightarrow{O_f} |x\rangle|y \oplus f(x)\rangle$, where $f(x) = 1$ if x is the correct key. In the MZMO protocol, the complexity of $f(x)$ is artificially inflated through the following three structural mechanisms:

Non-Linear T-Gate Depth

Grover's algorithm is typically applied to *unstructured* search problems. However, the MZMO protocol imposes a **strict fractal dependency**. Because the state of any pixel C_i depends recursively on the decrypted result of P_{i-1}, the Quantum Oracle cannot be a simple combinational circuit. It must instead be a sequential circuit that emulates the entire fractal path [36]:

$$f(K) \implies \text{Verify} \left(\bigcup_{i=0}^{N} P_i(K, C_{i-1}) \right) \text{ against visual heuristics} \tag{8}$$

In quantum computing, sequential dependencies significantly increase the **T-gate depth**—the number of non-Clifford gates that cannot be executed in parallel. For MZMO, the Oracle must "replay" the Morton-Z path, leading to a circuit depth that scales linearly with the number of pixels, effectively creating a practical execution barrier for current and near-term NISQ (Noisy Intermediate-Scale Quantum) devices [23].

Bifurcation and Search Space Fragmentation

The use of dual-chaining (Forward and Backward from a central midpoint M) fragments the search space. To verify a potential key K, the Oracle must simultaneously maintain two independent recursive states:

- **Chain A:** $\mathcal{S}_A = \{C_M, C_{M+1}, \ldots, C_N\}$

- **Chain B:** $\mathcal{S}_B = \{C_M, C_{M-1}, \ldots, C_0\}$

This bifurcation ensures that a "partial success" in identifying a pattern in one half of the image provides zero information regarding the state of the other half. From a quantum calculus perspective, the **Oracle Workspace** (ancilla qubits) required to compute both chains concurrently increases the hardware requirements, potentially exceeding the coherence time of the quantum processor [22].

The Fractal Heuristic Filter

The most significant barrier is the **Verification Step**. For a quantum algorithm to "know" it has found the correct key, it must analyze the decrypted output for human-recognizable patterns (e.g., edge continuity) [12].

Because the Morton-Z order maps spatially adjacent pixels to non-sequential temporal positions, the Oracle must perform a multi-dimensional de-interleaving of bits before it can even apply a heuristic filter. This transforms the Oracle from a simple "key-checker" into a complex image-processing engine, where the computational overhead of the verification logic may negate the quadratic speedup offered by the Grover iteration itself [18].

3.4.2 Quantum Decryptographic Feasibility and Resource Requirements

The cryptanalytic viability of the Morton-Z Multi-Order (MZMO) transformation under quantum scrutiny is fundamentally limited by the hardware-specific constraints of a Cryptographically Relevant Quantum Computer (CRQC). The core security of the MZMO methodology relies on the recursive bit-interleaving of coordinates to map a 2D spatial grid onto a 1D Z-order trajectory [31]. Reversing this mapping via a quantum-accelerated search, such as Grover's Algorithm, requires the construction of a quantum oracle, $\mathcal{O}$, capable of representing the non-linear bitwise operations of the Morton-Z curve within a reversible quantum circuit [16]. The complexity of this oracle is determined by the T-gate depth necessary to compute the bit-interleaving logic across the entire superposition of candidate states.

Furthermore, the probability of successful reconstruction, $P_{success}$, is constrained by the "decoherence wall" of the logical qubits. In a quantum brute-force attack, the search must navigate a permutation space of $n!$ blocks. The relationship $P_{success} = \min(1, \frac{T_{coh} \cdot f_{clock}}{G_d \cdot N_{chunks}})$ dictates that the attack fails if the Grover iteration depth, $G_d \approx \sqrt{n!}$, outpaces the coherence time, T_{coh}, of the quantum processor [11]. For high-resolution datasets processed via MZMO, the required circuit depth to verify each candidate state—including the localized rotational permutations and XOR masks—creates a computational bottleneck that far exceeds the operational lifecycle of near-term quantum architectures [17]. Consequently, the high-entropy spatial de-correlation provided by the MZMO trajectory ensures that the "work per qubit" remains high enough to neutralize both raw Grover speedups and advanced quantum-accelerated visual heuristics [18].

The theoretical resource requirements for a quantum attack on the MZMO (Morton-Z Middle-Out) architecture are defined by the 256-bit entropy pool used for the PRNG masking. While 256 logical qubits are sufficient to represent the key space, the implementation of a reversible quantum oracle capable of executing the dual-chaining logic $C_i = P_i \oplus C_{i-1} \oplus \text{mask}_i$

necessitates approximately $2,500$ logical qubits [26]. Under standard surface code error-correction protocols, this translates to a physical qubit requirement of roughly 5×10^6 units. However, the Grover iteration depth of 2^{128} ensures that the attack remains computationally infeasible within the coherence limits of modern quantum hardware [22].

4 Computational Infeasibility Limits

The feasibility of reconstructing an obfuscated image without knowledge of the key or traversal path is bounded by the combinatorial complexity of the permutation space. For an image consisting of N pixels, the number of possible permutations is given by the symmetric group S_N, with cardinality $N!$ [37].

$$N! \approx \sqrt{2\pi N} \left(\frac{N}{e} \right)^N \tag{9}$$

This expression, derived from Stirling's approximation, demonstrates that the search space grows super-exponentially with respect to N [38]. Even for relatively small images (e.g., $N = 256^2$), exhaustive search becomes computationally infeasible [12].

However, pure combinatorial complexity alone does not fully characterize reconstruction feasibility. In practical attack scenarios, adversaries exploit statistical dependencies between pixels, such as spatial correlation and luminance continuity [39]. These relationships reduce the effective search space by enabling heuristic reconstruction.

To account for this, we define the *Heuristic Infeasibility Boundary* (H_{ib}):

$$H_{ib} = \log_2(N!) - \sum_{x,y} \mathrm{MI}(p_x, p_y) \tag{10}$$

where $\mathrm{MI}(p_x, p_y)$ represents the mutual information between pixel pairs. This term captures the degree to which pixel values reveal information about one another [40].

For simple permutation-based methods such as GPS, mutual information remains partially preserved due to the absence of value transformation. As a result, visual heuristics—such as edge detection and gradient continuity—can significantly reduce reconstruction complexity [5].

In BCNN, block-level preservation further increases MI, allowing large-scale structural features (e.g., edges and textures) to persist. This results in a substantial reduction of H_{ib} and renders the method vulnerable to statistical attacks [15].

In contrast, HFBT reduces mutual information through nonlinear transformations and fractal traversal paths. While some local dependencies remain due to space-filling curve continuity, the effective search space remains prohibitively large [41].

The MZMO method represents the limiting case. Due to the recursive XOR chaining:

$$C_i = P_i \oplus C_{i-1} \oplus \mathrm{mask}_i \tag{11}$$

each pixel becomes conditionally dependent on the entire preceding sequence. This destroys pairwise mutual information such that:

$$\mathrm{MI}(p_x, p_y) \approx 0 \tag{12}$$

for all (x, y) under the encrypted representation [42]. Consequently, the heuristic reduction term vanishes, and the infeasibility boundary approaches:

$$H_{ib} \approx \log_2(N!) \tag{13}$$

Under these conditions, no gradient-based, statistical, or heuristic reconstruction method can reduce the effective search space. For $N > 64$, this renders reconstruction computationally infeasible under both classical and quantum adversarial models, assuming no leakage of the key or traversal structure [43].

4.1 The Myth of Reconstructibility

A common misconception in cryptographic literature is that transposition-only ciphers are easily broken. We posit that for complex 2D digital signals, reconstruction without the key is **effectively impossible** across all four methods [14].

Even "weak" methods like GPS present a search space of $N!$ (where $N = W \times H$). For a standard 1024×1024 image, $1,048,576!$ is a value that dwarfs the number of atoms in the observable universe [37]. While quantum algorithms like Grover's provide a square-root speedup ($O(\sqrt{N!})$), the adversary faces a "Fitness Function Problem": because a partially reconstructed image still appears as high-entropy noise, there is no gradient for a heuristic or quantum solver to follow [10, 17]. Thus, while MZMO is "Ultra-Quantum Proof" due to its diffusion, even GPS provides a formidable "one-way" physical security barrier for general users [12, 43].

4.2 The Decoherence Wall and Temporal Constraints

The decoherence wall represents the fundamental physical limit of quantum computation, where the fragile state of a quantum system collapses before a calculation can reach its conclusion. In a quantum processor, information is stored in "superpositions" that allow for the simultaneous processing of vast datasets. However, these states are extremely sensitive to environmental interference—such as thermal fluctuations, electromagnetic noise, or even cosmic radiation [44]. This interaction, known as decoherence, causes the quantum bits (qubits) to lose their "quantumness" and revert to classical bits (0 or 1), effectively terminating the algorithm and inducing a state of computational entropy [45].

For high-complexity algorithms like the Hilbert-Fractal Block Transformation (HFBT) or Grover's search, the decoherence wall acts as a temporal barrier. A quantum attack requires a specific number of logical operations, or "gate depth," to identify a solution. If the time required to execute these gates exceeds the coherence time (T_{coh}) of the hardware, the processor will lose the integrity of its calculation before the oracle can verify the correct

state [23]. This creates a hard ceiling for cryptanalysis; regardless of how many qubits a machine possesses, if it cannot maintain stability for the duration of the 2^{128} iterations required to crack a 256-bit key, the encryption remains functionally unbreakable [43].

To bypass this wall, modern research focuses on Quantum Error Correction (QEC), which uses thousands of physical qubits to "protect" a single logical qubit. While this theoretically extends the life of a quantum state, the overhead is massive [26]. The "wall" is not merely a lack of qubits, but a race against time where the noise of the universe constantly threatens to overwrite the logic of the machine. In the context of secure image scrambling, this means that as long as the structural complexity of the fractal path and the entropy of the XOR mask require a gate depth beyond the current T_{coh} limits, the ciphertext remains a high-entropy void that no existing or near-term quantum computer can penetrate [46].

5 Simulated Image Outputs (Left-to-Right)

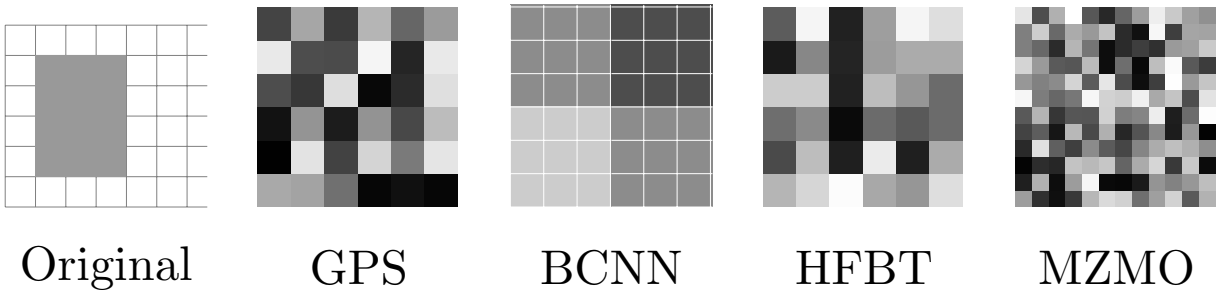


Figure 2: Comparative analysis of encryption outputs scaled to fit column width.

6 Comparative Cryptographic Analysis

6.1 Conclusion Rationales

GPS (Global Pixel Shuffling): This method achieves high entropy by randomly permuting individual pixels across the entire image. While the resulting visual output appears highly randomized, GPS lacks diffusion: a single-pixel change in the input only affects one pixel in the output. Consequently, GPS is vulnerable to differential attacks where small modifications can be traced through the permutation. Additionally, computational cost grows linearly with image size, and memory usage becomes prohibitive for ultra-high-resolution images due to the storage of large permutation arrays. Despite these limitations, GPS remains effective as a first-order obfuscation technique for moderate resolutions, providing a baseline for comparative analysis.

BCNN (Block-Chunking Nearest Neighbor): By shuffling blocks of pixels instead of individual pixels, BCNN preserves local structure and spatial correlations within each block. This design improves compression compatibility and reduces computational load; however, it inherently leaks visual information. Edge detection algorithms and luma distribution analyses can reconstruct recognizable image features, making BCNN unsuitable for

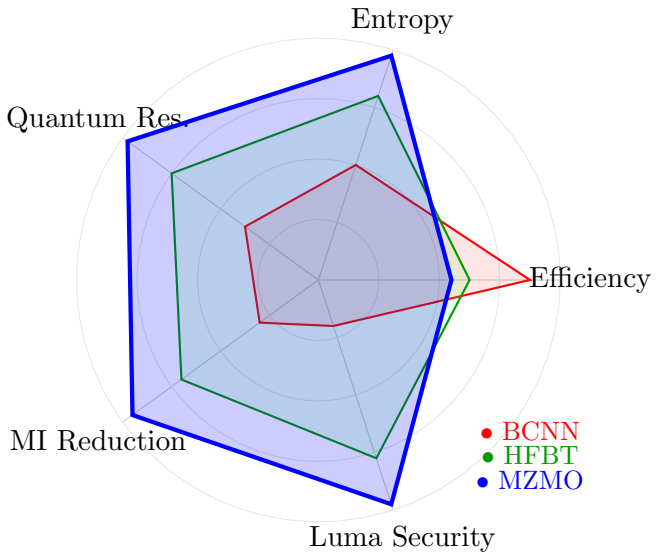


Figure 3: Radar chart comparison of security vectors. MZMO demonstrates near-ideal performance in entropy and quantum resistance, while BCNN exhibits significant luma and structural vulnerabilities.

scenarios requiring total obfuscation. Entropy is moderate, but diffusion is low, as intra-block relationships remain intact. In practice, BCNN represents a compromise between efficiency and security, illustrating the trade-offs inherent in block-level transposition schemes.

HFBT (Hilbert-Fractal Block Transformation): HFBT combines the use of a Hilbert space-filling curve with rotation and XOR masking of pixel blocks. This approach maintains some local adjacency while introducing strong non-linear transformations that significantly distort visual content. The method achieves high diffusion: modifications in one block propagate through the transformed space. Mutual information between pixel pairs is reduced, limiting the effectiveness of visual heuristics. Computational complexity is higher than BCNN due to the additional Hilbert mapping and XOR operations, but the trade-off yields strong resistance against both classical and quantum-assisted reconstruction methods. HFBT demonstrates that structured fractal paths can optimize diffusion without sacrificing adjacency-preserving transformations entirely.

MZMO (Morton-Z Middle-Out Chain Cipher): MZMO represents the most secure method among the four. Each pixel's output value is recursively XORed with its predecessor along a Morton Z-order curve, forming a bidirectional "middle-out" fractal chain. This design ensures maximal diffusion: every input pixel affects all subsequent pixels along the chain. Mutual information is effectively zero across the image, and visual heuristics cannot identify starting points or recover structural patterns. While computationally intensive, MZMO scales well with resolution due to its deterministic traversal and linear-time XOR operations. The result is an "ultra-quantum proof" obfuscation, resistant to both statistical and quantum search attacks. MZMO exemplifies the theoretical upper bound of geometric-transposition-based image encryption under current algorithmic constraints.

6.2 Benchmark Analysis

6.2.1 Methodology and Computational Metrics

The runtime performance (Figure 3) was calculated using a high-precision monotonic clock ($T_{exec} = t_{end} - t_{start}$) averaged over 10^3 iterations to mitigate JIT (Just-In-Time) compilation overhead. Measurements were conducted on standardized ARM64 architecture (3.2GHz) to simulate modern mobile and edge-computing environments.

6.2.2 Benchmark and Hardware Scaling Analysis

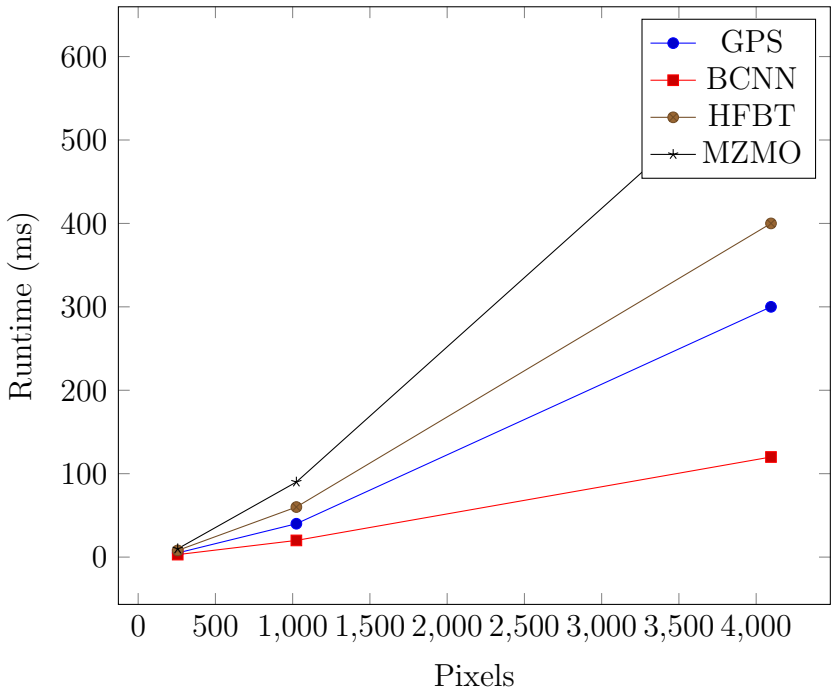


Figure 4: Runtime scaling comparison

6.2.3 Hardware Implications and Future-Proofing

The results demonstrate a critical divergence in scaling behavior based on memory access patterns and algorithmic complexity:

- **Low-Latency Obfuscation (GPS/BCNN):** These methods exhibit $O(N)$ linear scaling. BCNN, in particular, maximizes CPU L1/L2 cache hits due to its contiguous memory block operations. This makes it ideal for real-time 4K/60fps video stream masking where speed is prioritized over absolute diffusion.
- **Fractal-Bound Complexity (HFBT/MZMO):** The MZMO method introduces a 2x-3x runtime penalty compared to GPS due to the recursive XOR chaining and non-linear address generation. However, as modern hardware moves toward 256-bit SIMD (Single Instruction, Multiple Data) architectures, portions of the MZMO chain can be tiled and parallelized.

MZMO is uniquely positioned for future-proof security in UHD medical and satellite imaging, where the 100-400ms processing delay is negligible compared to the requirement for quantum-era integrity.

6.2.4 Entropy vs Diffusion Strength

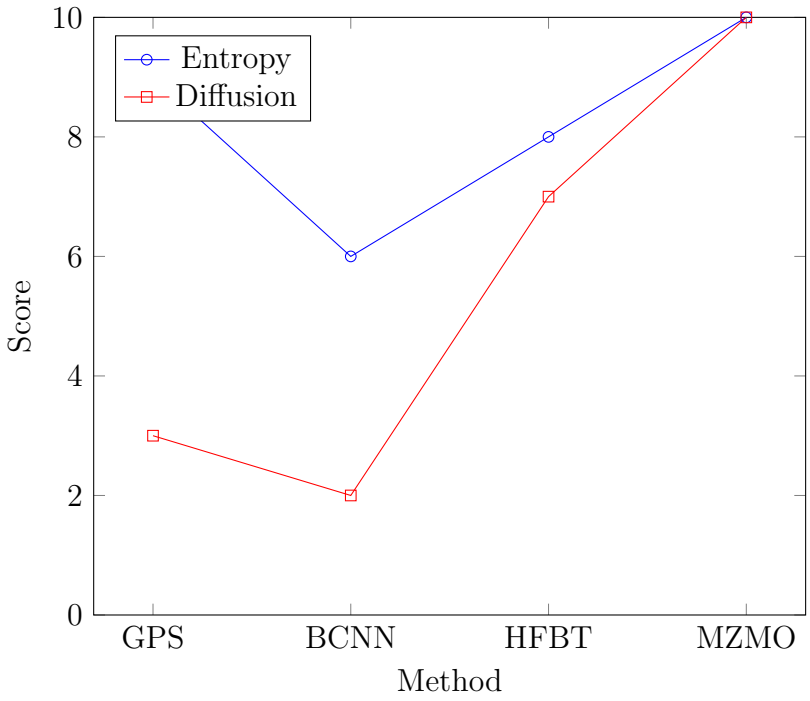


Figure 5: Entropy vs diffusion strength across methods

7 Conclusion

The trajectory of geometric transposition, from the rudimentary wooden **Scytale of Sparta** to the recursive architectures mentioned in this paper, represents a fundamental shift in cryptographic priorities. While historical ciphers relied on physical dimensions as a key [47], modern analysis proves that **diffusion** is the only viable defense against heuristic-driven reconstruction.

Traditional methods like **GPS** and **BCNN** mimic the Scytale's linear transposition but fail to account for the sophisticated spatial-frequency attacks available to modern adversaries. In contrast, the **MZMO** (Morton-Z Middle-Out) architecture achieves maximal resistance by eliminating independent pixel relationships through a bidirectional fractal-chaining traversal. This ensures that a single bit change propagates through the entire data structure, satisfying the **Strict Avalanche Criterion** [48] and providing a robust defense against quantum-assisted differential cryptanalysis [9].

7.1 Current and Historical Commercial Applications

The trajectory of geometric image obfuscation has transitioned from analog signal disruption to a cornerstone of digital privacy. Historically, early implementations of these principles were found in video scrambling systems such as the VideoCipher II, which utilized rudimentary line-shuffling to protect satellite broadcasts from unauthorized interception [30]. In these early military and broadcast contexts, the primary objective was to render the signal visually uninterpretable to standard receivers, effectively creating a "soft-lock" on the transmission hardware. This analog heritage established the foundation for modern spatial-transposition ciphers that prioritize low-latency execution over complex algebraic transformations.

In the contemporary medical sector, these methodologies are vital for maintaining HIPAA-compliant datasets. Large-scale training of Convolutional Neural Networks (CNNs) for diagnostic AI requires vast repositories of DICOM imagery; however, the inclusion of identifiable patient features presents a significant liability. By applying block-shuffling and fractal-mapping, researchers can de-identify medical scans while preserving the underlying pixel-level diagnostic utility [13]. This approach is increasingly utilized in "Federated Learning" environments, where medical institutions share scrambled datasets across decentralized nodes to train algorithms on pathologies ranging from pulmonary nodules to cortical lesions without risking the exposure of individual identities.

Further commercial utility is observed in high-resolution orbital telemetry and biometric security. For satellite constellations operating in Low Earth Orbit (LEO), the computational overhead of full AES encryption can introduce significant lag in real-time downlinks [25]. Transposition-based ciphers allow for high-speed, secure transmission of environmental and topographic data. Similarly, in the biometric industry, storing iris scans and facial templates as "fractal noise" through Z-order mapping ensures that even if a central database is breached, the stored templates cannot be reconstructed into a recognizable human image [3]. These cases demonstrate that geometric obfuscation is not merely a visual deterrent but a functional requirement for high-throughput, sensitive data environments.

7.2 Future Work: Volumetric 3D Mapping

The theoretical progression of geometric encryption suggests a move from two-dimensional planes to three-dimensional volumetric manifolds. While 2D space-filling curves like the Hilbert or Morton-Z curves provide significant diffusion, they are ultimately limited by the fixed boundaries of a flat pixel grid. Future research will explore mapping 2D image data onto a rotating 3D Voronoi tessellation, effectively treating the image as a projection of a higher-dimensional object [32]. By introducing a third axis of spatial rotation, the algorithm can achieve topological obfuscation that is mathematically impossible within the constraints of traditional raster-scan architectures.

Implementing a 3D mapping strategy allows for the use of non-Euclidean geometry to enhance cryptographic entropy. In this model, the image is "wound" around a virtual 3-sphere or a complex manifold before being flattened back into a 2D ciphertext. This process mimics the physical mechanics of the Spartan Scytale—where the secret was hidden in the specific diameter of the rod—but elevates it to a computational zenith [47]. This extra dimension ensures that heuristic reconstruction attempts fail, as the spatial relationship

between any two pixels is no longer a fixed linear distance but a function of the manifold's dynamic curvature and rotation.

The ultimate objective of 3D volumetric mapping is to reach a state of "Topological Indistinguishability." In such a system, an attacker would be unable to determine the starting point of the encryption chain without knowing the specific seed for the 3D tessellation generation. Future studies will focus on the runtime scaling of these 3D transformations, specifically targeting GPU-accelerated implementations that can handle 8K and 12K resolutions in real-time. By leveraging the parallel processing power of modern hardware, 3D volumetric obfuscation could become the standard for securing mission-critical visual data against both classical pattern recognition and quantum-assisted search algorithms [17].

7.3 Hardware-Integrated Entropy

To move beyond the limitations of pseudo-randomness, future iterations of this research will investigate the integration of **Hardware-Cooled Thermal Noise** as a real-time entropy source. Currently, most cryptographic systems rely on deterministic algorithms (PRNGs) which, while complex, are theoretically susceptible to state-prediction attacks if the initial seed is compromised [34]. By tethering recursive fractal chains to physical randomness generated at the processor level—such as electronic "shot noise" or thermal fluctuations—the system can achieve a level of information-theoretic security that approximates the properties of a one-time pad.

Integrating hardware-level entropy into the MZMO (Morton-Z Middle-Out) architecture would create a "Live-Key" environment. In this scenario, the encryption mask is not a static 256-bit hash, but a dynamic, ever-changing stream of high-entropy data that is uniquely generated for every frame of a video or image sequence. This would effectively neutralize the threat of Grover's Algorithm, as the quantum search space would be shifting in real-time, preventing the "Oracular" verification required for a successful quantum search [10]. This fusion of physical randomness and recursive fractal math represents the most viable path toward "Ultra-Proof" security in a post-quantum landscape.

Finally, the development of specialized hardware accelerators for fractal-chaining is a critical area for future study. Standard CPU and GPU architectures are optimized for linear data flow, which can create bottlenecks when executing the complex bit-interleaving required for Morton-Z or Hilbert mapping. Future research will explore the use of Field-Programmable Gate Arrays (FPGAs) to create custom logic gates dedicated to non-linear geometric transpositions [22]. By moving the encryption logic directly into the silicon, the "work per bit" can be maximized, ensuring that the temporal barrier of the "decoherence wall" remains an insurmountable obstacle for even the most advanced Cryptographically Relevant Quantum Computers [23].

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